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Three-tangle rate controlled by local operation

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Abstract

Since local unitary operations do not change any entanglement measures, to manipulate the distribution of multipartite entanglement, non-local unitary operations should be required. Given a three-qubit system, we study the change of three-tangle under bipartite unitary evolution. We find, once the form of a bipartite interaction is fixed, the changing rate of three-tangle depends on additional single-qubit unitary operations. Furthermore, we give an explicit definition of the three-tangle rate controlled by local operations. By taking advantage of this three-tangle rate, we discuss three examples.

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1. Introduction

Entanglement is one of the central topics of quantum science [1]. It shows some fantastic behavior that is not presented in classical physics. The generation of entanglement often involves global unitary operation. To quantify the capability of creating entanglement for the global unitary operation, a lot of effort [2–6] has been devoted to the entanglement power of two party unitary operators. Particularly, an entanglement power of the two-qubit global unitary applied on bipartite pure states is given by Zanardi *et al* [2].

Although local operations do not change entanglement [7], they could serve as a control to manipulate the entanglement behavior under global evolution. Dür defines the entanglement rate of two qubits under non-local evolution as $\mathrm{d}E/\mathrm{d}t$ [8] and controls the rate by local operations. Jordan *et al* control some bipartite and tripartite entanglement measures of the three-qubit mixed state $\rho_{AB} \otimes \frac{1}{2} 1_C$ under a non-local Hamiltonian by local operation [9].

Here we focus on a new issue: consider a three-qubit pure state $|\Psi\rangle_{ABC}$ which evolves under a two-party unitary operation $U_{BC}(t)$. Taking three-tangle as a genuine entanglement measure, we investigate how the three-party genuine entanglement changes at the beginning of the evolution. We use the changing rate to quantify the Hamiltonian's capability of changing entanglement. Concretely, we apply a local operation U_C on qubit C before the global unitary evolution $U_{BC}(t)$. It is worth noting that the entanglement behavior of the system will change under the new unitary $U_{BC}(t)(1_B \otimes U_C)$. By manipulating U_C, we can control the entanglement-changing rate and make the process of entanglement increasing or decreasing more efficient. However, according to Dür's definition $\mathrm{d}E/\mathrm{d}t$ (E is the entanglement measure), we find that $\frac{\mathrm{d}\tau}{\mathrm{d}t}\big|_{t=0^+}$ cannot describe the entanglement change of some special but important classes of the state, such as GHZ state, W state and biseparable states. We have to modify Dür's definition $\mathrm{d}E/\mathrm{d}t$ and find other available functions to describe the entanglement-changing rate.

In this paper, we give a new definition of the three-tangle rate to solve this problem. We choose a maximal time interval $t \subset (0, t^*)$ during which the three-tangle changes monotonically for all evolutions. Then we could quantify their capability of changing genuine entanglement by observing the three-tangle at $t = t^*$. As will be shown in examples 1 and 2, for GHZ state, the three-tangle rate under the XXZ model could be maximized by rotating the qubit C about the y-axis. But for W state, we have to consider all the parameters of U_C to maximize the three-tangle rate. In example 2, we will extend our method by applying a local unitary on different qubits to optimize the three-tangle rate. In example 3, we find that, for the biseparable initial state $|\Psi\rangle_{AB} \otimes |0\rangle_C$, the three-tangle is directly related to the concurrence of $|\Psi\rangle_{AB}$. We can obtain the three-tangle at any time of the evolution by simply measuring the concurrence of $|\Psi\rangle_{AB}$. Lastly, by taking advantage of the three-tangle rate, we relate the generation of the three-tangle to the total entanglement resource of the initial system.

This paper is arranged as follows. In section 2, we first review the concept of the concurrence and three-tangle. Then we will give an example to illustrate that local operation could strongly affect the entanglement behavior of the three-qubit system under global evolution. In the central part of this section, a new definition of the three-tangle rate controlled by local operation will be given. In section 3, we will discuss some representative examples.

2. The three-tangle rate controlled by U_C

Before embarking on our central issue, let us review some important entanglement measures. There have been many entanglement measures for two qubits, such as the entanglement of formation [14] and relate entropy of entanglement [15]. Wootters *et al* established the concurrence as the measure of a general two-qubit mixed state [16].

Consider a general two-qubit state with the density matrix ρ, the concurrence of ρ is given by

$$C = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\},$$

where λ_i's are the eigenvalues of $\rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$ in decreasing order.

The residual entanglement [10] of the three-qubit pure state is defined as the three-tangle: $\tau = C^2_{A|BC} - C^2_{AB} - C^2_{AC}$, here C is the concurrence of the two-party subsystem. The three-tangle τ of the pure state $|\Psi\rangle_{ABC} = \sum_{i,j,k=0}^{1} a_{ijk}|ijk\rangle$ could be expressed by the coefficients:

$$\tau = |d_1 - 2d_2 + 4d_3|,$$

where

$$\begin{aligned} d_1 &= a_{000}^2 a_{111}^2 + a_{001}^2 a_{110}^2 + a_{010}^2 a_{101}^2 + a_{100}^2 a_{011}^2; \\ d_2 &= a_{000}a_{111}a_{011}a_{100} + a_{000}a_{111}a_{101}a_{010} \\ &\quad + a_{000}a_{111}a_{110}a_{001} + a_{011}a_{100}a_{101}a_{010} \\ &\quad + a_{011}a_{100}a_{110}a_{001} + a_{101}a_{010}a_{110}a_{001}; \\ d_3 &= a_{000}a_{110}a_{101}a_{011} + a_{111}a_{001}a_{010}a_{100}. \end{aligned}$$

Dür *et al* have shown that the three-qubit pure state could be classified by two SLOCC inequivalent classes [11]: the GHZ and W classes. In this work, we choose $|\mathrm{GHZ}\rangle = (1/\sqrt{2})(|0\,0\,0\rangle + |1\,1\,1\rangle)$ and $|\mathrm{W}\rangle = (1/\sqrt{3})(|0\,0\,1\rangle + |0\,1\,0\rangle + |1\,0\,0\rangle)$ as representatives for these two classes and choose τ to measure the tripartite genuine entanglement. For the GHZ class, the three-tangle $\tau \neq 0$; for the W class, $\tau = 0$. It implies that the GHZ class is genuine entangled but the W class has no genuine entanglement. In this paper, we restrict our discussion to the genuine entanglement.

The change of multipartite entanglement often involves global operation, in which the two-qubit unitary operation is a simple class. Let us consider a three-qubit system which evolves under a two-qubit unitary $U_{BC}(t)$. We apply a local operation U_C before the evolution. Although the local operation cannot affect the entanglement nature of the initial state, it is easily seen that the behavior of the entanglement will change under the new global unitary $U_{BC}(t)(1_B \otimes U_C)$. We give an example to illustrate this point of view.

Let the general three-qubit pure state be expanded by the standard basis: $|\Psi\rangle_{ABC} = \sum_{i,j,k=0}^{1} a_{ijk}|ijk\rangle$. Consider that $|\Psi\rangle_{ABC}$ evolves under the Ising unitary $U_{BC} = \exp\left(-\mathrm{i}\sigma_z^B \otimes \sigma_z^C t\right)$. By a direct calculation, it could be verified that U_{BC} is equivalent to local operation if anyone of the following four conditions is satisfied:

$$a_{0\,0\,0} = a_{1\,0\,0} = 0, \qquad a_{0\,0\,1} = a_{1\,0\,1} = 0,$$
$$a_{0\,1\,0} = a_{1\,1\,0} = 0, \qquad a_{0\,1\,1} = a_{1\,1\,1} = 0.$$

It is easy to see that both GHZ and W states belong to this class. Therefore, all the five local invariants of three qubits $(tr\rho_A^2, tr\rho_B^2, tr\rho_C^2, tr(\rho_A \otimes \rho_B \rho_{AB}), \tau_{ABC})$ [17] remain constant during the evolution.

If the third party applies an arbitrary unitary U_C, the GHZ state becomes $\alpha|0\,0\,0\rangle + \beta|0\,0\,1\rangle + \gamma|1\,1\,0\rangle + \delta|1\,1\,1\rangle$ and the W state becomes $\alpha|0\,0\,0\rangle + \beta|0\,0\,1\rangle + \gamma|0\,1\,0\rangle + \delta|0\,1\,1\rangle + \zeta|1\,0\,0\rangle + \eta|1\,0\,1\rangle$. Obviously, they no longer belong to the class, then U_{BC} is no longer equivalent to local operation. In this case, the local invariants will change correspondingly. In particular, the evolution becomes capable of changing the three-tangle τ_{ABC} after the operation U_C. This is apparently different from the Zanardi's entanglement power which is LU invariant [2, 3].

Several preview works have also focused on the entanglement change of the three-qubit state. Acin *et al* found the optimal single-copy local protocol [12] to distill a GHZ state from any three-qubit pure state with some probability by local operations. On the other hand, the generation of entanglement often involves global operation. By considering two-qubit global operation, Cai *et al* discussed the entanglement complementary behavior of the three-qubit product state $|\phi\rangle_A \otimes |0\rangle_B \otimes |0\rangle_C$ [13]. Jordan *et al* investigated the three-part entanglement of the three-qubit mixed state $\rho_{AB} \otimes \frac{1}{2}1_C$ under a simple unitary U_{AC} and took the qubit C as a control [9].

In this paper, we approach this problem in another perspective by studying the three-tangle changing rate. Let a three-qubit pure state be driven by a two-qubit unitary evolution:

$$|\Psi(t)\rangle = U_{BC}(t)|\Psi'(0)\rangle,$$

where $U_{BC}(t) = \exp\left[-\mathrm{i}\left(a\sigma_x^A \otimes \sigma_x^B + b\sigma_y^A \otimes \sigma_y^B + c\sigma_z^A \otimes \sigma_z^B\right)t\right]$. Here we change the initial state $|\Psi(0)\rangle$ by applying a local operation U_C on qubit C before the evolution: $|\Psi'(0)\rangle = U_C|\Psi(0)\rangle$, with

$$U_C(\theta,\gamma,\beta) = \mathrm{e}^{\mathrm{i}\alpha}\begin{pmatrix} \mathrm{e}^{-\mathrm{i}(\beta+\gamma)/2}\cos\dfrac{\theta}{2} & -\mathrm{e}^{-\mathrm{i}(\beta-\gamma)/2}\sin\dfrac{\theta}{2} \\ \mathrm{e}^{\mathrm{i}(\beta-\gamma)/2}\sin\dfrac{\theta}{2} & \mathrm{e}^{\mathrm{i}(\beta+\gamma)/2}\cos\dfrac{\theta}{2} \end{pmatrix},$$

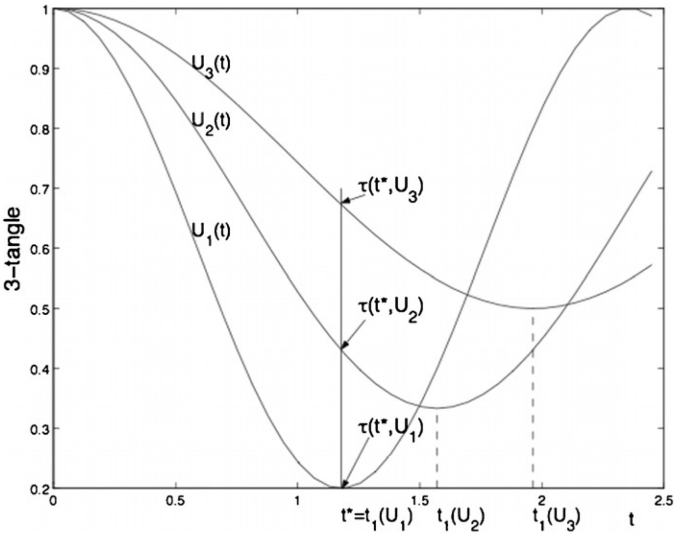

Figure 1. Three-tangle rate controlled by U applied on qubit C. First, we search all the U_C and obtain the corresponding evolutions. Then we find out the first extreme points t_1 of $\tau(t, U_C)$ under each evolution and denote the minimal t_1 by t^*(in this figure, $t^* = t_1(U_1)$). For a given U, we then obtain $R(U) = |\tau(t, U)_{t=t^*} - \tau(|\Psi(0)\rangle)|/t^*$ by observing the three-tangle of this evolution at $t = t^*$, i.e., the point of intersection of $t = t^*$ and all curves of $\tau(t, U)$.

where β and γ correspond to the rotation about the z-axis and θ corresponds to a rotation about the y-axis [7]. We can always neglect the phase factor $\mathrm{e}^{\mathrm{i}\alpha}$, as it is irrelevant to the three-tangle.

Hereby, for given $U_{BC}(t)$, the whole unitary $U^{\text{global}}(t) = U_{BC}(t)(1_B \otimes U_C)$ is determined by U_C. We can parameterize the three-tangle under $U^{\text{global}}(t)$ as $\tau(t, U_C)$. By manipulating U_C, we can promote the efficiency of changing three-tangle for the given $U_{BC}(t)$.

If it is desired to find U_C which maximizes the change of three-tangle when the evolution begins, it may appear as if we could choose $\mathrm{d}\tau(t, U_C)/\mathrm{d}t|_{t=0^+}$ to describe the rate. However, as will be shown in example 1, we find that $\mathrm{d}\tau(t, U_C)/\mathrm{d}t|_{t=0^+}$ is not an ideal function for some special (but important) states. For the states whose three-tangle are zero, such as the W state class and the separable state, $\mathrm{d}\tau(t, U_C)/\mathrm{d}t|_{t=0^+}$ is infinite. For the GHZ state, $\mathrm{d}\tau(t, U_C)/\mathrm{d}t|_{t=0^+}$ equals zero. We have to find other available functions to describe the change of three-tangle for these important classes.

It is a natural idea to investigate the three-tangle after a short interval Δt during which the three-tangle changes monotonically. For every $\tau(t, U_C)$, the monotonous interval differs correspondingly. Our goal is to obtain a monotonous interval $t \subset (0, t^*)$ for all $\tau(t, U_C)$. Then we can observe the three-tangle at $t = t^*$ of all the evolutions to quantify their capability of changing entanglement.

We search all the U_C to obtain all the corresponding evolutions. Then we can find out the first extreme value point $t_1(U_C) > 0$ of each $\tau(t, U_C)$ such that $\frac{\mathrm{d}\tau(t,U_C)}{\mathrm{d}t}\Big|_{t=t_1} = 0$ to ensure that $\tau(t, U_C)$ is monotonous at $t \subset (0, t_1)$. Denoting the minimal $t_1(U_C)$ of all evolutions by t^*(see figure 1), it turns out that $(0, t^*)$ is the maximal monotonous interval for all $\tau(t, U_C)$. Note that t^* is determined by the parameters a, b, c of $U_{BC}(t)$ and the initial state $|\Psi(0)\rangle$.

After the value of t^* is obtained, we search all the U_C again and observe the three-tangle change by averaging τ over a timescale t^* with respect to each $U^{\text{global}}(t)$. Then we could have a reasonably accurate estimate of the three-tangle rate controlled by local operation.

From the above discussion, we could define the three-tangle rate controlled by local operation mathematically: given a unitary U, the three-tangle rate $R(U)$ controlled by U is defined as

$$R(U) = \frac{|\tau(t, U)_{t=t^*} - \tau(|\Psi(0)\rangle)|}{t^*}, \tag{1}$$

here $t^* = \min_{U_C}(t_1)$, t_1 is the minimal positive number satisfying $\mathrm{d}\tau(t, U_C)/\mathrm{d}t|_{t=t_1} = 0$.

3. Examples

In this section, we will apply $R(U)$ to three particular examples. To proceed, we simplify $U_{BC}(t)$ by considering the XXZ model ($a = b \neq c$). Since we are more interested in the choice of U which maximizes $R(U)$ than the explicit value of $R(U)$, we can set $a = b = 1$ for simplicity.

Example 1. The GHZ state
After U_C, the GHZ state becomes

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(\mathrm{e}^{-\mathrm{i}(\beta+\gamma)/2}\cos\frac{\theta}{2}|0\,0\,0\rangle + \mathrm{e}^{\mathrm{i}(\beta-\gamma)/2}\sin\frac{\theta}{2}|0\,0\,1\rangle \right.$$
$$\left. - \mathrm{e}^{-\mathrm{i}(\beta-\gamma)/2}\sin\frac{\theta}{2}|1\,1\,0\rangle + \mathrm{e}^{\mathrm{i}(\beta+\gamma)/2}\cos\frac{\theta}{2}|1\,1\,1\rangle\right).$$

Under the evolution $U^{\mathrm{global}}(t)$, the three-tangle of $|\Psi\rangle$ after some time t becomes

$$\tau(t, U_C) = \left|\cos 4ct + \tfrac{1}{4}(1 - \cos 2\theta)(\cos 4t - \cos 4ct) + \cos\theta\,\sin(4ct)\mathrm{i}\right|. \tag{2}$$

To illustrate that definition (1) is necessarily required, let us examine Dür's definition of the entanglement rate $\mathrm{d}E/\mathrm{d}t$[8]. Let $\tau(t, U_C) = |z(t)|$, on differentiation we get

$$\frac{\mathrm{d}\tau(t, U_C)}{\mathrm{d}t} = \frac{\mathbf{Re}\, z(t)\frac{\mathrm{d}\,\mathbf{Re}\, z(t)}{\mathrm{d}t} + \mathbf{Im}\, z(t)\frac{\mathrm{d}\,\mathbf{Im}\, z(t)}{\mathrm{d}t}}{\sqrt{|z(t)|}}. \tag{3}$$

Combining equation (2) with equation (3), we arrive that, for the GHZ state, $\mathrm{d}\tau(t)/\mathrm{d}t|_{t=0^+}$ equals zero as $\mathbf{Im}\, z(t)|_{t=0^+} = 0$ and $\mathrm{d}\,\mathbf{Re}\, z(t)/\mathrm{d}t|_{t=0^+} = 0$. (By a similar procedure, it could be shown that even $a \neq b$, $\mathrm{d}\tau(t, U_C)/\mathrm{d}t|_{t=0^+} = 0$ still holds.) Moreover, it is easy to see that for the initial states whose three-tangle are zero (W class, biseparable class), $\mathrm{d}\tau(t, U_C)/\mathrm{d}t|_{t=0^+}$ is infinite. Consequently, $\mathrm{d}\tau(t, U_C)/\mathrm{d}t|_{t=0^+}$ cannot describe the change of entanglement for these states.

Let us return now to discuss our example. As equation (2) shows, $\tau(t, U_C)$ is independent of β and γ, hence it could be parameterized as $\tau(t, \theta)$. Then we could obtain the value of t^* by minimizing the first extreme values of $t_1(\theta)$. Figure 2 shows t^* versus the Hamiltonian parameter c. As an illustrative example, we set $c = 0.6$. Numerically, we get $t^* = 0.5$ for this evolution. Substituting t^* into equation (2), we obtain $\tau(t, \theta)_{t=t^*}$ and the three-tangle rate $R(U)$. As shown in figure 3, $R(U)$ reaches its maximum for $\theta = \left(k + \frac{1}{2}\right)\pi$, $\forall\,\beta$ and γ, $k \geqslant 0$, when $\tau(t, \theta)_{t=t^*}$ reaches its minimum.

Example 2. In this example we consider the state

$$|\Psi\rangle = \alpha_1|0\,0\,1\rangle + \alpha_2|0\,1\,0\rangle + \alpha_3|1\,0\,0\rangle. \tag{4}$$

Under the $U_{BC}\,(t)$ of the XXZ model, the three-tangle of this state can also be calculated and is given by

$$\tau(t, U_C) = 8\alpha_1^2\alpha_2\sin^2\frac{\theta}{2}\left|\alpha_2(\cos 4t - \cos 4ct)\cos^2\frac{\theta}{2}\right.$$
$$\left. - .\alpha_3\sin(\beta+\gamma)\sin 4t + \mathrm{i}\alpha_3\cos(\beta+\gamma)\sin 4t\right|.$$

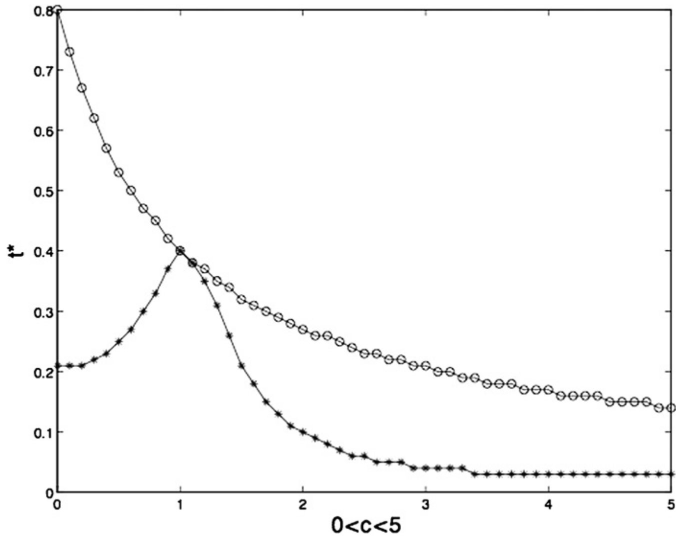

Figure 2. The range of t^* of GHZ (-o- line) and W (+ line) state for the XXZ model. We can take $c > 0$ because when we replace c by $-c$, the three-tangle of both GHZ and W states remain invariant. This is easily seen from their expressions (2) and (4).

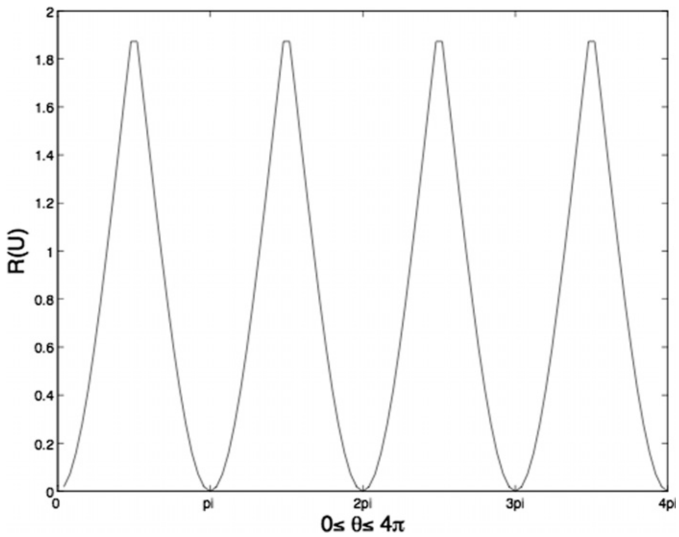

Figure 3. The range of $R(U)$ as a function of θ. GHZ state, $a = b = 1$, $c = 0.6$.

We will discuss this example in two points of view.

(1) We first discuss the W state by setting $\alpha_1 = \alpha_2 = \alpha_3 = \frac{1}{\sqrt{3}}$. By a similar calculation to the GHZ state, we can also obtain t^* (see figure 2) for fixed c and the three-tangle rate $R(U)$. For $c = 0.6$, we obtain $t^* = 0.27$. By setting $\varphi = \beta + \gamma$, we parameterize $\tau(t, U_C)$ as $\tau(t, \theta, \varphi)$. The corresponding three-tangle rate $R(t, \theta, \varphi)_{t=t^*}$ is depicted graphically as in figure 4. In fact, by a numerical calculation, it could be concluded that

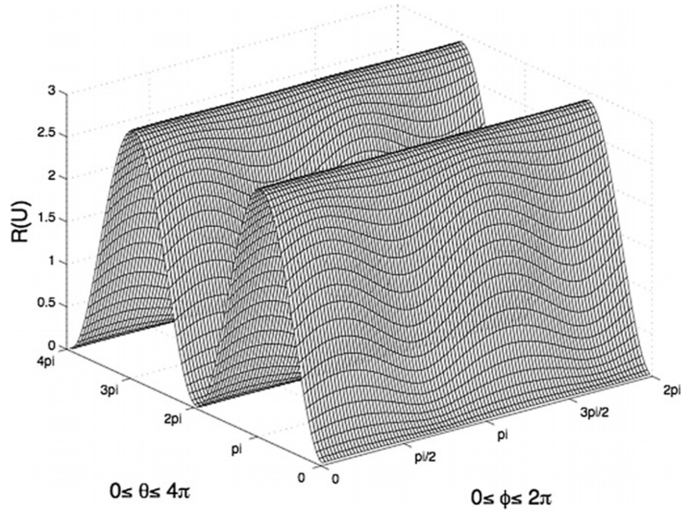

Figure 4. The distribution of $R(U)$ of the W state if $a = b = 1$, $c = 0.6$.

if $0 \leqslant c \leqslant 4.24$, $R(U)$ is maximal for $\theta = (2k+1)\pi$, $k \geqslant 0$, $\forall\,\beta$ and γ. We only need to rotate the initial state about the y-axis to reach this maximum. Otherwise, if $c > 4.24$, we have to consider the parameters β and γ, which correspond to the rotation about the z-axis. It should be emphasized that, if we observe the three-tangle of the system at t', here $t' > t^*$, the distribution of the maximal value of $\tau(t,\theta,\varphi)|_{t=t'}$ becomes very complex and differs completely from $\tau(t,\theta,\varphi)|_{t=t^*}$. This apparent contradiction arises because $\tau(t,\theta,\varphi)|_{t=t'}$ cannot accurately describe the entanglement generation during the interval $t \subset (0, t')$. This confirms the importance of our definition (1).

(2) In fact, we could extend our method by applying three local unitaries U_A, U_B and U_C on all the three qubits to extract the optimal rate. However, this general case involves complicated calculation as all the nine parameters of the three unitaries must be considered. Here, we apply a single unitary on each qubit of $|\Psi\rangle$ respectively, and then compare the three results of the optimal rate. By a similar calculation we find that $\tau(t, U_A) = 0$ and

$$\tau(t,U_B) = 8\alpha_1^2\alpha_3\sin^2\frac{\theta}{2}\left|\alpha_3(\cos 4t - \cos 4ct)\cos^2\frac{\theta}{2} - \alpha_2\sin(\beta+\gamma)\sin 4t + \mathrm{i}\alpha_2\cos(\beta+\gamma)\sin 4t\right|.$$

We choose $\alpha_1 = \alpha_3 = \frac{1}{2}, \alpha_2 = \frac{1}{\sqrt{2}}$ and set $c = 0.6$ again. From the expression of $\tau(t, U_B)$ and $\tau(t, U_C)$, we obtain their $t^* = 0.31$ and 0.22 respectively. In both cases, the three-tangle rate reaches its maximum at $\theta = (2k+1)\pi$ where $R(U_B) = 2.16$ and $R(U_C) = 2.48$. Hence, we get a higher maximal three-tangle rate if we apply the local unitary on qubit C.

Example 3. Another example is provided by the biseparable state $|\Psi\rangle_{AB} \otimes |0\rangle_C$, here only the first two qubits are entangled and the third one is not. Let the initial state $|\Psi(0)\rangle = (a|0\,0\rangle + b|0\,1\rangle + c|1\,0\rangle + d|1\,1\rangle) \otimes |0\rangle_C$, we obtain

$$\tau(t,U_C) = 4|ad - bc|^2\sin^2\theta\,|\cos 4t - \cos 4ct|. \tag{5}$$

Proceeding as before, we can also obtain $R(U)$ and then maximize it by manipulating U. In this example, we discuss the question in some other views. From equation (5), it is interesting to note that, as the concurrence of the subsystem AB of the initial state is $2|ad - bc|$, $\tau(t, U_C)$ is proportional to C_{AB}^2. Denoting $f(t, c) = |\cos 4t - \cos 4ct|$, we could rewrite the right-hand side of equation (5) as $C_{AB}^2 \sin^2\theta f(t, c)$. Thus, we separate $\tau(t, U_C)$ into three parts: C_{AB}^2 (the entanglement of the initial state), $\sin^2\theta$ (the effect of the local operation) and the factor $f(t, c)$ (a function of t and the global operation parameter c). In an experimental realization, it is much easier to detect the concurrence of the two-qubit pure state [19, 20] than to detect the three-tangle. In our example, we can obtain the three-tangle at any time during the evolution by simply detecting the concurrence of the arbitrary initial state $|\Psi\rangle_{AB}$.

Moreover, it is worth noting that the total entanglement resource of the initial state is just the bipartite entanglement of AB as qubit C is separable. By applying fixed $U_{BC}(t)$ on different separable states $|\Psi\rangle_{AB} \otimes |0\rangle_C$, we have the same resource of physical operation, the same structure of the state, but a different entanglement resource. Equation (5) implies that, for fixed $U_{BC}(t)$, when the total entanglement resource of the state increases, the three-tangle rate $R(U)$ rises correspondingly. This result may increase our understanding of the process of entanglement generation. It is reasonable to believe that the generation of the three-tangle is not only dependent on the global operation, but also related to the entanglement resource of the initial system.

4. Conclusion

By defining $R(U)$, we have given a detailed description of the three-tangle rate controlled by local unitary operation under a two-qubit unitary evolution $U_{BC}(t)$. We provide a method for investigating the change of a quantity which cannot be described by differentiation. We find the local operations which maximize the rate for several examples under the XXZ model. We also extend our method by applying local unitary on different qubits of an asymmetric state to optimize the three-tangle rate. For the biseparable state $|\Psi\rangle_{AB} \otimes |0\rangle_C$, the three-tangle rate rises with the increase of entanglement of the initial state. The three-tangle at any time during the evolution could be obtained by measuring AB's concurrence of the initial state.

However, we still lack a lucid description about the mechanism of multipartite entanglement transformation. How the entanglement resource is shared during the global evolution, whether and how the bipartite entanglement could be transformed into tripartite entanglement are still open questions.

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